① TDC part III Paper V Group A Mathematical physics

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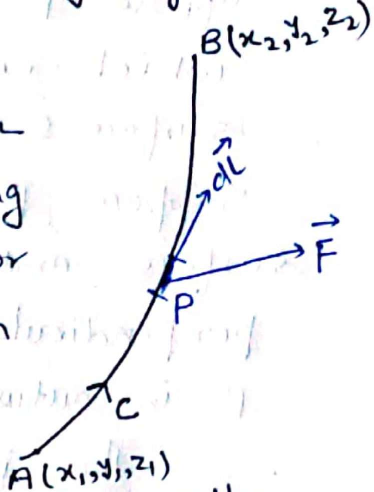
Line Integral, Surface Integral & Volume Integral

* **Line Integral:-** The integral evaluated along any curve, is known as Line integral.



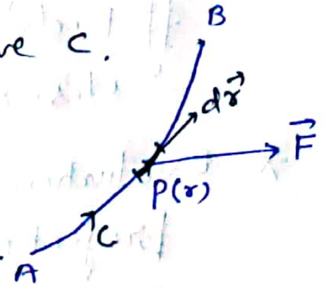
Suppose $\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$ is a vector point function which is continuous along the curve C. The integral of the vector point function $\vec{F}$ along the curve C from A to B is denoted by $\int_A^B \vec{F}\cdot\vec{dl}$ and $\int_A^B \vec{F}\cdot\vec{dl}$ is known as Line integral of the vector point function $\vec{F}$ along the curve C from A to B.

Now $\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$ and $\vec{dl} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$$\int_A^B \vec{F}\cdot\vec{dl} = \int_A^B (F_1\hat{i} + F_2\hat{j} + F_3\hat{k})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$= \int_{x_1}^{x_2} F_1\,dx + \int_{y_1}^{y_2} F_2\,dy + \int_{z_1}^{z_2} F_3\,dz$$

Note :- The line integral around a simple closed curve is denoted by integral sign with a circle around it as $\oint$ in place of $\int$ and the line integral $\oint_C \vec{F}\cdot\vec{dl}$ is known as circulation of $\vec{F}$ about closed curve C.



Use of line integral: work done by a force $\vec{F}$:

Let a force $\vec{F}$ acts upon a particle which displaces it from A to B along a given curve C.

Total work done by the force $\vec{F}$ is $W = \int_A^B \vec{F}\cdot\vec{dr}$

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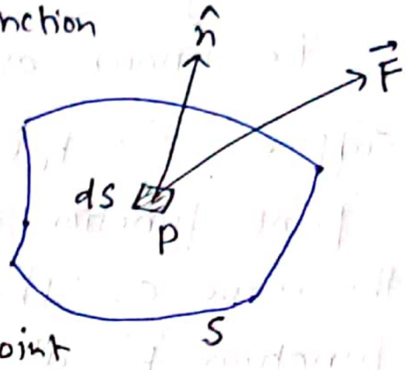
* Surface integral :- The integral which is evaluated over a surface, is known as surface integral.

Suppose $\vec{F}$ be a vector point function which is continuous over the surface S.

Suppose P be any point on the surface and $\hat{n}$ is a unit vector perpendicular to the surface at point P in outward direction.



The integral of the vector point function $\vec{F}$ over the surface S is denoted by $\iint_S \vec{F}\cdot\hat{n}\,ds$ or $\iint_S \vec{F}\cdot d\vec{s}$ where $d\vec{s} = \hat{n}\,ds$ and the integral $\iint_S \vec{F}\cdot\hat{n}\,ds$ is known as surface integral or normal surface integral of the vector point function $\vec{F}$ over the surface S.

$d\vec{s} = \hat{n}\,ds$ = vector area of small surface around P directed outward and normal to the surface at P.

Note :- The surface integral over a closed surface is denoted by integral with a circle around it as $\oint_s$ or $\oiint_S$ in place $\int_s$ or $\iint_S$ and the surface integral $\oiint_S \vec{F}\cdot d\vec{s}$ is known as surface integral of $\vec{F}$ over a closed surface S.

* Evaluation of surface integral with the help of orthogonal projection of the surface S on one of the coordinate plane :- Surface integral can be easily evaluated by expressing it as double integral taken over orthogonal

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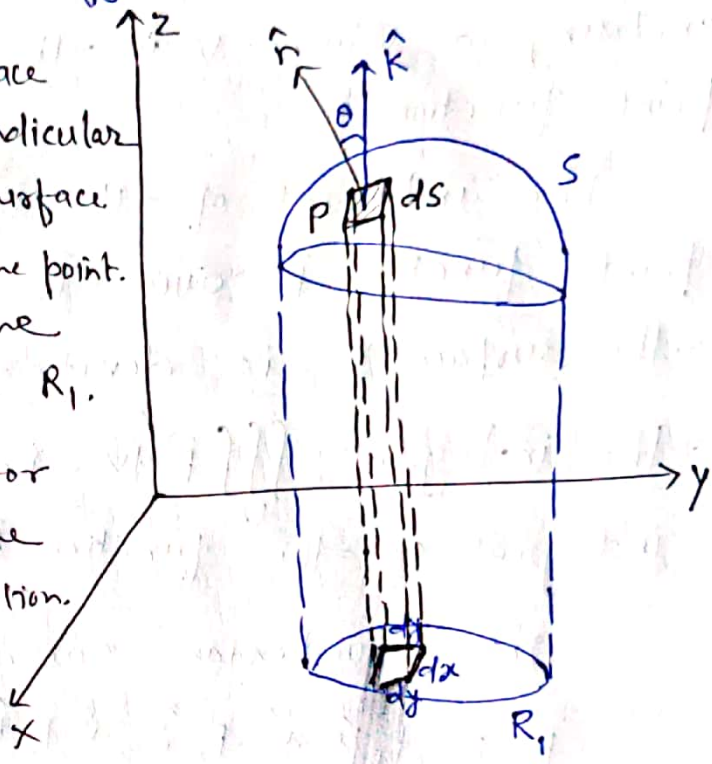
projection of the surface S on one of the coordinate planes. But this is possible only when any line perpendicular to choosen coordinate plane meets the surface S not more than one point. If a surface does not satisfy this condition then it can be subdivided into surfaces which satisfy the condition.

Suppose S be a surface such that any line perpendicular to xy plane meets the surface S ~~non~~ not more than one point. let the projection of the surface S on xy plane is R_1.

Let $\hat{n}$ be a unit vector at point P normal to the surface in outward direction. and it makes an angle θ with $\hat{k}$.



$$d\vec{s} = \hat{n}\, ds$$

component of $d\vec{s}$ along z axis $= ds.\cos\theta = dx.dy$

$$\Rightarrow ds = \frac{dx.dy}{\cos\theta}$$

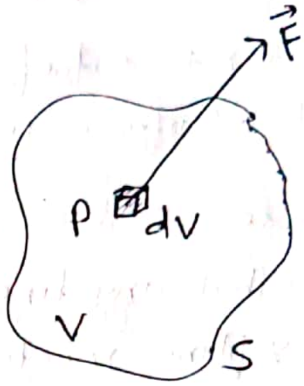
Now $\hat{n}.\hat{k} = |\hat{n}|.|\hat{k}|.\cos\theta = 1.1.\cos\theta \Rightarrow \cos\theta = \hat{n}.\hat{k}$

$$\Rightarrow ds = \frac{dx.dy}{|\cos\theta|} \Rightarrow ds = \frac{dx\,dy}{|\hat{n}.\hat{k}|}$$

Now surface integral $\iint_S \vec{F}.\hat{n}\,ds = \iint_{R_1} \vec{F}.\hat{n}.\frac{dx.dy}{|\hat{n}.\hat{k}|}$

* **Volume integral:-** The integral which is evaluated over a volume enclosed by a surface in vector point function, is known as volume integral.

Suppose $\vec{F}$ be a vector point function and a surface S is enclosing a volume V in the vector point function $\vec{F}$.



The integral of the vector point function $\vec{F}$ over the volume V enclosed by the surface S is denoted by $\iiint_V \vec{F}\, dV$ and the integral $\iiint_V \vec{F}\, dV$ is known as volume integral of the function $\vec{F}$ over the volume V.

In Cartesian coordinate system

$$\vec{F} = F_1 \hat{i} + F_2 \hat{j} + F_3 \hat{k} \quad \text{and} \quad dV = dx\,dy\,dz$$

$$\text{Volume integral } \iiint_V \vec{F}\, dV = \iiint_V (F_1 \hat{i} + F_2 \hat{j} + F_3 \hat{k})\, dx\,dy\,dz.$$

Where F_1, F_2 and F_3 may be functions of x, y and z.